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Waveforms and Spectrograms

Sound is generated by a periodic disturbance of molecules. For example, when a musician plucks the D-string on a guitar, the string vibrates at a particular frequency dictated by the material of the string and its tension. The string pushes on the air molecules around it. In turn, the air molecules push on their neighbors. A sound wave is an oscillating pattern of compression and rarefaction of molecules in response to some force. If the waves from the guitar reach a person's ear, biochemical processes allow the person to hear the waves as the musical note known as D3.

What makes one sound D and another A? This is a somewhat elusive question, but one simple answer is speed. When our musician played the D-string, the string began completing about 145 cycles of compression and rarefaction per second. This is known as 145 Hertz. If the musician were to play the next string, A, it would vibrate at about 110 Hertz. The frequency of a sound dictates its pitch (whether the sound is high or low).

Another important aspect of sound is intensity. This is a result of the amplitude of a sound, essentially how far molecules move from equilibrium in response to the sound. Intensity can be measured in decibels. A normal conversation is usually about 60 decibels, while the music

at a loud rock concert can reach as high as 120 decibels. The intensity of a sound determines how loud the sound is perceived to be.

Consider figure 1, which displays waveform diagrams of three different versions of the same note, A4 (440 Hz). A waveform graphs the intensity of the sound in decibels with respect to time. The first is of a pure tone generated by a computer. Observe that it appears the sound stays



Figure 1. These are waveforms of a 1.5 second note played at 440 Hz. The first was made by a computer, the second by a saxophone, and the third by a piano.

at the same intensity throughout. Next comes the note as played by a saxophone; below that, by a piano. Both experience damping, a gradual lessening of intensity with time, although at different rates. This should be no surprise when one considers the mechanics of these instruments: As soon as the saxophonist stops blowing, as soon as the hammer comes off the piano string, the vibration begins to decrease in strength.

The waveforms above are somewhat misleading. Although they appear to be solid, it is only because they display 1.5 seconds at once. Zooming in reveals more instantaneous behavior,

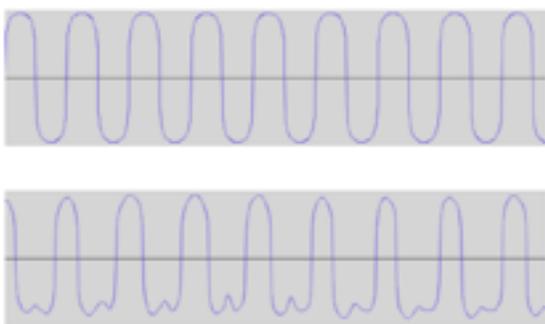


Figure 2. This is a closer view of the first and third waveforms from figure 1, each covering about a twentieth of a second.

as can be seen in figure 2. Observe that the pure tone at the top, while obviously not always at the same intensity, is much more uniform than the piano at the bottom. Nevertheless they are relatively similar in appearance, especially in amplitude and period.

The series of images in figure 3 display spectrograms for each of these same recordings. A spectrogram charts the spectral density of a sound with respect to time. Spectral density refers to the energy the sound has at particular frequencies at a particular time. It is determined from the time signal using the short-time Fourier transform, which approximates the sound using a series of complex amplitudes. The amplitudes are approximated by the coefficients of the short-time Fourier transform.

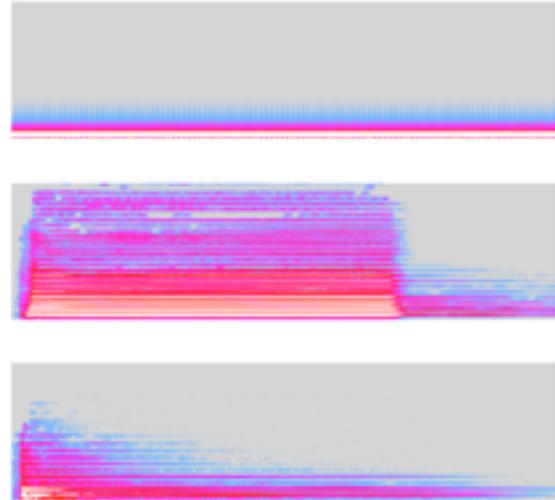


Figure 3. These are spectrograms of the same recordings used to make each of the waveforms found in figure 1.

In this process, linear algebra is used to decompose the sound signal into a linear combination of the basis. The basis is made up of predetermined basic sounds that can be combined to form all other sounds. Assuming the signal is in the space spanned by the basis, you are able to choose the basis in a particular way that gives rise to a complementary orthonormal basis called a dual function. By taking the inner product of the linear combination and the dual function, the result is the coefficients used in the short-time Fourier transform.

Observe that the shape of the sound in the waveform, a result of the nature of the force causing the sound, is preserved here. However, these spectrograms reveal something not shown in the waveforms. We see the pure tone has its energy concentrated below 5000 Hz, while the saxophone has moderately intense energy at frequencies higher than 15,000 Hz, even though the both are playing the same 440 Hz note. Further, as long as the note is being played, there very

high-intensity energy at the lowest frequencies, visible as the white stripe. When playing stops, the energy at these levels quickly decreases.

Advances in modern technology allow people to analyze sound with far greater accuracy and power than was ever possible before. While intuition would dictate that three instruments playing the same note would create identical sound waves, these waveforms and spectrograms make it obvious that there are apparently more distinctions than similarities. How will future technology further enlighten us about the nature of sound?